

# Winter Workshop on FSI problems

January 10-11-12 2024

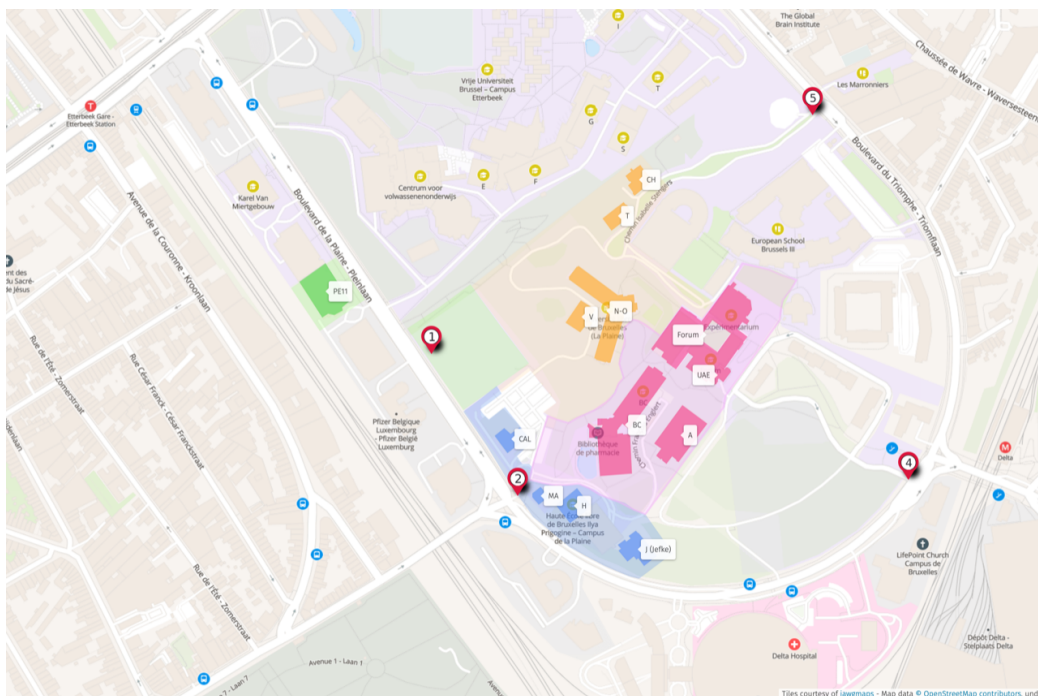
Université libre de Bruxelles

INFOS & PROGRAM

## Practical information

The conference will take place in the Building NO of the Campus Plaine on the 9th floor in the Room N0906. The Apart-hotel is located Opposite The Chirec Delta Hospital Entrance, Blvd du Triomphe 207, it is a 10' walk to the conference Room. On the map below, UAE (Union des anciens étudiants) is the place where we will have the social dinner and the lunch on Thursday. FORUM is the place you can have your own lunch on Wednesday and Friday.

- Closest Train station is ETTERBEEK.
- Closest Metro station is DELTA.
- Closest Bus stop is FRAITEUR.



## Wednesday 10

14h15-14h55

**Olivier Glass (Université Paris-Dauphine)**

**Small solids in Euler flows**

We consider the evolution of rigid bodies in a perfect incompressible fluid. The fluid is driven by the incompressible Euler equation, while the solids evolve according Newton's equations under the pressure force on their boundary. We investigate the question of the limit of the system as (some of) the solids converge to a point while keeping a constant velocity circulation on their boundary. We obtain a complete picture when each solid can belong to either of the three categories:

- solids that have a fixed size,
- solids that shrink to a point while keeping a fixed mass,
- solids that shrink to a point and having mass going to zero.

In the limit, we obtain a system coupling the Euler equation for the fluid, Newton's equations for the non-shrinking bodies, and massive/non-massive point vortices for the remaining ones. This is a joint work with Franck Sueur, following works with Christophe Lacave, Alexandre Munnier and Franck Sueur.

15h05-15h45

**Thierry Gallay (Université Grenoble Alpes)**

**Dynamics of vortex pairs in two-dimensional viscous fluids**

As a model for the viscous interaction of planar vortices, we consider the solution of the two-dimensional Navier-Stokes equation with singular initial data corresponding to a pair of point vortices with equal or opposite circulations. In the large Reynolds number regime, we construct an approximate solution which takes into account the leading order correction to the translation or rotation speed of the vortex centers, due to finite size effects. As a consequence of enhanced dissipation, our approximation remains valid over increasingly long time intervals as the viscosity parameter goes to zero. The proof relies on stability estimates derived from Arnold's variational characterization of the steady states in the 2D Euler equation. This talk is based on joint work with M. Dolce (EPFL).

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– Cookies and Chocolates –

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16h15-16h55

**Clara Patriarca (Université Libre de Bruxelles)**

**Long-time behavior of an anisotropic rigid body interacting with a Poiseuille flow in an unbounded 2D channel**

We study the long-time behavior of an elliptic rigid body which is allowed to vertically translate and rotate in a 2D unbounded channel under the action of a Poiseuille flow at large distances. The motion of the fluid is modelled by the incompressible Navier-Stokes equations, while the motion of the ellipse is described through Newton's laws in the presence of additional elastic restoring forces but without any structural damping. Our contributions are a global-in-time existence result and

a proof of return to equilibrium. Both results require to rule out a possible collision between the body and the channel boundaries. To prove global-in-time existence, we show that the distance between the ellipse and the channel walls remains uniformly positive for all times if the flow rate is below a first threshold. This no-collision result requires new fine estimates due to the necessity of a uniform control in time and the possible local asymmetry of the collision configuration. It is well-known that the viscosity of the fluid gives a dissipation on the kinetic energy of the body but no direct dissipation on its potential energy. Also, the hydrodynamic force and torque split into two competing components: one that, indeed, forces the oscillations of the body while the other contribution tends to absorb them, due to viscosity effects. To prove asymptotic stability of the equilibrium, we show that if the flow rate is below a second threshold, the fluid viscosity brings enough dissipation on the potential energy of the body.

**17h05-17h45**

**Pei Su (Charles University)**

**Regularity of the system describing an elastic beam interacting with the Navier-Stokes equations**

We consider the interaction between a beam and the incompressible Navier-Stokes equations. The fluid equations are coupled with the beam via the kinematic condition and the action-reaction principle on the interface. The beam is assumed to be perfectly elastic, which thereby gives a hyperbolic evolution. We show the new regularity result for this parabolic-hyperbolic coupled system. It turns out that the parabolic effect of the fluid suffices to regularize the solution to the coupled fluid-structure system which is previously known for the Navier-Stokes equations in fixed domains.

This is a joint work with S. Schwarzacher (Uppsala and Prague).

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– Belgian Beers and other drinks at 6:30 pm at UAE –

– Social Dinner at 7:30 pm at UAE –

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**Thursday 11**

**9h15-9h55**

**Ewelina Zatorska (Imperial College London)**

**On the multi-dimensional extension of the Aw-Rascle model**

I will discuss the extension of the Aw-Rascle model to multi-dimensional case and present our recent results on that model concerning: existence and uniqueness of solutions, and their asymptotic limits in the “hard congestion regime”.

10h05-10h45

**Charlotte Perrin (Université d'Aix-Marseille)**

**Hard congestion limit of the p-system in the BV setting**

In this talk, I will discuss the transition from a compressible (inviscid) system with singular pressure towards a mixed compressible-incompressible system modeling partially congested dynamics. The two systems may be used for the modeling of mixtures, of collective motions, or partially free surface flows. From the mathematical point of view, I will present a first convergence result for small BV perturbations of a reference state represented by one or more partially congested propagating fronts.

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– Coffee & Tea –

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11h15-11h55

**David Lannes (Université de Bordeaux)**

**The 2D nonlinear shallow water equations with a partially immersed obstacle**

This talk (based on a joint work with T. Iguchi) is devoted to the proof of the well-posedness of a model describing waves propagating in shallow water in horizontal dimension  $d=2$  and in the presence of a fixed partially immersed object. We first show that this wave-interaction problem reduces to an initial boundary value problem for the nonlinear shallow water equations in an exterior domain, with boundary conditions that are fully nonlinear and nonlocal in space and time. This hyperbolic initial boundary value problem is characteristic, does not satisfy the constant rank assumption on the boundary matrix, and the boundary conditions do not satisfy any standard form of dissipativity. Our main result is the well-posedness of this system for irrotational data and at the quasilinear regularity threshold. In order to prove this, we introduce a new notion of weak dissipativity, that holds only after integration in time and space. This weak dissipativity allows high order energy estimates without derivative loss; the analysis is carried out for a class of linear non-characteristic hyperbolic systems, as well as for a class of characteristic systems that satisfy an algebraic structural property that allows us to define a generalized vorticity. We then show, using a change of unknowns, that it is possible to transform the linearized wave-interaction problem into a non-characteristic system, which satisfies this structural property and for which the boundary conditions are weakly dissipative. We can therefore use our general analysis to derive linear, and then nonlinear, a priori energy estimates. Existence for the linearized problem is obtained by a regularization procedure that makes the problem non-characteristic and strictly dissipative, and by the approximation of the data by more regular data satisfying higher order compatibility conditions for the regularized problem. Due to the fully nonlinear nature of the boundary conditions, it is also necessary to implement a quasilinearization procedure. Finally, we have to lower the standard requirements on the regularity of the coefficients of the operator in the linear estimates to be able to reach the quasilinear regularity threshold in the nonlinear well-posedness result.

12h05-12h45

Paolo Galdi (University of Pittsburgh)

On Self-Propulsion by Oscillations in a Viscous Liquid

Let  $\mathcal{B}$  be a body moving in an otherwise quiescent Navier-Stokes liquid,  $\mathcal{L}$ , that fills the entire space outside  $\mathcal{B}$ . We will consider the case where  $\mathcal{B}$  is prevented from performing rigid rotations around its center of mass  $G$ , a condition that can be realized by applying a suitable torque on  $\mathcal{B}$ .

Denote by  $\Omega = \Omega(t)$ ,  $t \in \mathbb{R}$ , a one-parameter family of bounded, sufficiently smooth domains of  $\mathbb{R}^3$ , each one representing the configuration of  $\mathcal{B}$  at time  $t$  with respect to a frame with the origin in  $G$ . We assume that there are no external forces acting on the coupled system  $\mathcal{S} := \mathcal{B} \cup \mathcal{L}$  and that the only driving mechanism is a *prescribed* change in shape of  $\Omega$  with time, in a given precise way.

The self-propulsion problem that we would like to address can be thus qualitatively formulated as follows. Suppose that  $\mathcal{B}$  changes its shape in a given time-periodic fashion, so that, for some  $T > 0$  and all  $t \in \mathbb{R}$ ,  $\Omega(t+T) = \Omega(t)$ . Then, find sufficient conditions on the map  $t \mapsto \Omega(t)$  securing that  $\mathcal{B}$  self-propels, namely, the center of mass  $G$  covers any given finite distance in a finite time.

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– Lunch at UAE –

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14h15-14h55

Laurel Ohm (University of Wisconsin - Madison)

Free boundary dynamics of an elastic filament in 3D Stokes flow

We consider a free boundary problem for a thin elastic filament immersed in 3D Stokes flow. The 3D fluid is coupled to the quasi-1D filament dynamics via a novel type of angle-averaged Neumann-to-Dirichlet operator. Much of the difficulty in the analysis lies in understanding this operator. We show that the principal part of this NtD map is the corresponding operator about a straight, periodic filament, for which we derive an explicit symbol. It is then possible to establish local well-posedness for an immersed filament evolving via a simple elasticity law.

15h05-15h45

Filippo Gazzola (Politecnico di Milano)

Symmetry and stability in fluids and structures

We discuss the relationship between symmetry and stability in both fluids and structures. We show that in fluid-structure interactions, symmetry appears essential for stability, since lift forces exist only if non-symmetric solutions exist. Moreover, symmetry-breaking of the solutions may lead to instability and this seems to be related to symmetry-breaking of some Sobolev minimizers. We also present a situation in which the solutions of an evolution Navier–Stokes problem converge to the steady symmetric solution for large times. As for the sole structure, we show that a symmetric isolated two-span beam may be more prone to develop instability than an asymmetric one. Thus, symmetry may be disadvantageous from a purely structural point of view. These results are obtained with M. Garrione and with several other collaborators.

**16h15-16h55**

**Takeo Takahashi (Institut Élie Cartan de Lorraine)**

**The Prodi-Serrin condition for weak solutions of a fluid-rigid body interaction system**

This talk is devoted to the properties of the weak solutions of a fluid-structure interaction system if we assume the Prodi-Serrin condition. Our system describes the motion of a rigid ball in a viscous incompressible fluid and we assume that the fluid-rigid body system fills the entire space. We show that the Prodi-Serrin condition yields the uniqueness and the regularity of weak solutions.

**17h05-17h45**

**Paolo Zunino (Politecnico di Milano)**

**Fluid-structure interaction of slender bodies immersed in three-dimensional flows**

In collaboration with Muriel Boulakia, Céline Grandmont, Miguel-Angel Fernández, Fabien Lespagnol.

We consider the simulation of slender structures immersed in a three-dimensional flow. Exploiting the special geometric configuration of the slender structures, this problem can be modeled as coupling equations defined on domains with different dimensions: three dimensions for the fluid and one dimension for the structure. This modeling approach can be formulated in the framework of mixed-dimensional equations. Several challenges must be faced when dealing with such problems. From a mathematical point of view, these include defining discrete trace operators of codimension two. On a computational standpoint, the non-standard mathematical formulation makes it difficult to ensure the convergence of the discrete model and to assess the accuracy of the solutions obtained with the mixed-dimensional discrete model as compared to a fully resolved (3D) model.

We establish the continuous formulation using the Navier-Stokes equations for the fluid and a Timoshenko beam model for the structure. We complement these models with a mixed-dimensional version of the fluid-structure interface conditions, based on the projection of kinematic coupling conditions on a well-chosen finite-dimensional approximation space. Furthermore, we develop a discrete formulation within the framework of the finite element method, establish the energy stability of the scheme, provide numerical evidence of the convergence of the discrete formulation, and assess its accuracy with respect to a fully resolved Arbitrary Lagrangian-Eulerian (ALE) model. In this way, the proposed formulation solves the difficulties described above. Eventually, we show the robustness of the method by considering multiple beams subjected to a Navier-Stokes flow with a moderate Reynolds number.

**Friday 12**

**9h15-9h55**

**Karoline Disser (Universität Kassel)**

**Global solutions and non-trivial long-time behaviour for fluid-elastic interaction with small data**

Joint work with Michelle Luckas (Universität Kassel).

We look at a geometrically linearized but non-linear system modelling the dynamics of a linearly elastic body immersed in an incompressible viscous fluid in 3d. We show that with no other damping but fluid viscosity, but with small initial data, the system admits a unique global strong solution that converges either to a steady state or a “pressure wave”-solution. We show the existence of non-trivial pressure waves for particular geometries of the elastic structure. The non-existence of pressure waves for a large class of domains follows from previous work of Avalos and Triggiani.

**10h05-10h55**

**Daniel Han Kwan (Université de Nantes)**

**Local wellposedness of a Vlasov equation for bubbly flows**

We discuss the well-posedness of a Vlasov equation, formally derived by Herrero, Lucquin-Desreux and Perthame as the mean-field limit of dispersed bubbles in a potential flow. We prove that the system is locally well-posed under an (optimal) stability condition for the initial condition. This is a joint work with Kleber Carrapatoso (Ecole polytechnique) and Frédéric Rousset (Université Paris Saclay).

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– Tea & Coffee –

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**11h15-11h55**

**Aline Lefebvre-Lepot (CNRS-CentraleSupélec)**

**Numerical simulation of suspensions: taking close interactions into account**

We address the problem of direct numerical simulation of suspensions made of rigid particles in Stokes flow. We focus on singular effects due to short-range interactions (lubrication effects) as the particles approach each other. Taking these lubrication effects into account in numerical simulations is a difficult problem: capturing the singularity requires, for example, the use of very fine meshes in the gap between particles. Here, we describe two methods for taking lubrication effects into account numerically. The first is based on an asymptotic expansion of the solution in the particle gap. It enables accurate results to be obtained while using coarse meshes and without adding new assumptions or models. We then describe a second method, based on a “viscous” contact model. From a numerical point of view, it reduces to solving a convex optimization problem at each time step. It will be shown that dry contacts, with or without friction, can be treated within the same theoretical framework. Finally, the coupling problem between these different contact models will be discussed.

12h05-12h45

**Thomas Richter (Otto-von-Guericke-Universität Magdeburg)**  
**FSI with elastic contact - Numerical Modeling and Benchmarking**

We consider an elastic solid, e.g. a ball, which sinks towards the ground in a container filled with a liquid. The ball bounces off the bottom.

From a mathematical point of view, it is not clear what exactly happens. Assuming that the Navier-Stokes equations describe the situation, contact should not occur. The common hypothesis is that if the surface of the sphere is assumed to be perfectly smooth, a thin film of liquid always remains, the forces are transferred into the solid body via this film and are released again due to the elasticity. This results in a rebound without any contact.

In the lecture, the numerical difficulties of simulating such a problem are discussed. The various common approaches, e.g. ALE coordinates or purely Eulerian formulations, all have their own challenges to overcome.

We show that a simulation is possible without adding parameterized additional models to describe the rebound. However, this requires an immense effort. In addition, we present experimental and numerical benchmark problems as well as computational results.